

Math 261 - Final Exam
Los Angeles City College - Spring 2008

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Name _____

1. Find the following limit: $\lim_{x \rightarrow 0} \left(\frac{1}{x} \right) \left(\frac{1}{\sqrt{1+x}} - 1 \right)$
2. Find the following limit: $\lim_{x \rightarrow 0} \left(\frac{\sin(2x) - x}{\tan x} \right)$
3. Prove that the following function is continuous for all real numbers: $f(x) = \sqrt{\frac{1+x^2}{\sin x + 2}}$
4. Find $f'(x)$ if $f(x) = \sqrt{\frac{x^2-2x}{1-x^3}} (2x-3)^4$
5. Find $f'(x)$ if $f(x) = \cos^5(4 \sin^3 2(\tan(x^2 - 1)))$
6. Find $f'(x)$ if $f(x) = \sqrt{\sin \sqrt{\cos \sqrt{x}}}$
7. Find y' for $\sqrt{x^2 - y^2} - \sin y = \cos^2 x - 5y$
8. Find the rate of change of hypotenuse of a right triangle as a function of time, if when $x = 3$, $y = 4$, $\frac{dy}{dt} = 1.5$ & $\frac{dx}{dt} = -2$
9. Graph one period of the following function $f(x) = \sin 2x - \cos x = 0$. Find 1st. & 2nd derivatives, critical as well as inflection points and all intercepts.
10. Use the following graph of $f'(x)$ to construct a possible sketch of $f(x)$

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12. A metal cylindrical container with an open top is to hold one cubic foot. If there is no waste in the construction, find the dimension that requires the least amount of material.
13. A force of 25 pounds is needed to stretch a spring from its natural length of 12 inches to 13 inches. Find the work needed to stretch this spring from the 13 inches to 15 inches.
14. The end of an 8 foot long container are equilateral triangles with the sides of length of 2 ft. If this container is filled with water, calculate the work needed to empty this container.

15. The ends of a 6 ft long water container have the shape of the region bounded by $y = x^2$ & $y = 4$ when x and y are measured in ft. If this container is filled with water, find the force exerted by the water on one end?